

The University of Chicago

AN INVESTIGATION OF THRUST FAULTING

A DISSERTATION SUBMITTED TO THE
FACULTY OF THE DIVISION OF THE
PHYSICAL SCIENCES IN CANDIDACY FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF GEOLOGY
AND PALEONTOLOGY

1937

By
ELWOOD ATHERTON

Private Edition, Distributed by
THE UNIVERSITY OF CHICAGO LIBRARIES
CHICAGO, ILLINOIS

1940

The University of Chicago

AN INVESTIGATION OF THRUST FAULTING

A DISSERTATION SUBMITTED TO THE
FACULTY OF THE DIVISION OF THE
PHYSICAL SCIENCES IN CANDIDACY FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF GEOLOGY
AND PALEONTOLOGY

1937

By

ELWOOD ATHERTON

Private Edition, Distributed by
THE UNIVERSITY OF CHICAGO LIBRARIES
CHICAGO, ILLINOIS

1940



INTRODUCTION

In this paper some of the problems of thrust faulting are approached both by experimental and by mathematical methods. The mathematical treatment is in four parts. The first part deals with the motion of a rigid block on a plane surface; the second, with the motion of a rigid block on a curved surface; the third, with the elastic motion of a flat plate; and the fourth, with the effect of bending in an elastic block. The results described in this last section proved helpful in explaining some of the phenomena observed in the experiments. The advice and criticism of Professor R. T. Chamberlin were very helpful in this work.

EXPERIMENTAL METHODS

In the experimental study of thrust faulting, blocks of wax were deformed in a simple pressure machine. The particular purpose of these experiments was to investigate the structures produced by the movement of fault blocks. In each experiment two blocks of wax were placed together, as in Figure 1, and one pushed

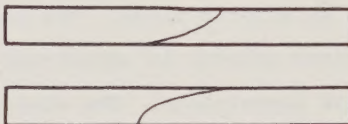


Fig. 1.--Position of blocks

over or under the other. Although the forces that cause a major fracture will, no doubt, form minor structures at the same time or slightly earlier, and these primary minor structures will influence the secondary structures made after the blocks begin to move, it seemed advisable, for the sake of simplicity, to start the experiments with the major fracture already present. Otherwise the secondary structures produced by the slip on the fault and continuing pressure can not readily be separated from the minor primary structures which develop just before or at the time of the formation of the major fracture. This method has other advantages. First, the shape of the initial fracture can be controlled; it can be made convex or concave up, and the effects of this difference determined. Arbitrary assumptions as to the attitude of major fractures in the earth are thus in part avoided, and any analogies between structures made experimentally and those

found in the rocks may be more directly applied in the field. Second, by placing the major fracture near the middle of the mass, secondary structures can be formed at some distance before or behind this fracture. If this fracture were near the piston of the pressure machine, secondary structures could be formed back only as far as the piston, and their nature might be influenced considerably by the rigidity of the piston.

There is some difficulty in choosing the material to be used in experiments such as these. Pieces of actual rock are generally unsuitable. In addition to this difficulty, the time involved in the deformation of the specimen can be only a minute fraction of the time during which deformative forces may act on rocks in nature. Previous investigators have usually chosen some weak material such as plaster or wax, and they have in a short time produced structures simulating those found in the field which were produced by forces acting for a long time. Though this resemblance is in some respects more apparent than real, it is close enough for such experiments to be instructive.

In this investigation hard and soft waxes were used. The hardness and strength of the wax were roughly proportional; the harder wax was more brittle than the softer, but even the soft wax fractured readily. Some of the wax used had about the consistency of paraffin; at the other extreme was wax, made soft by the addition of axle grease, which had about the consistency of butter. The blocks of wax deformed in the experiments were about two inches thick, eight inches wide, and each about twelve inches long. Some blocks were massive; others were of layers of equal hardness, and still others were of layers of unequal hardness. In the process of preparation, after one layer had cooled, the hot wax for the next layer was poured over the cool layer. In this

way the layers were made to adhere to each other. The wax was cast in a rectangular trough with removable bottom. At each end of the trough was placed a plaster of Paris mold (Fig. 2). Wooden

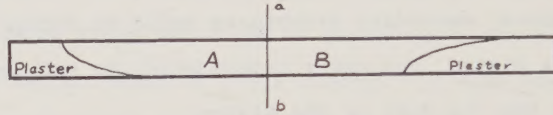


Fig. 2.--Mold for wax

templates were used to give the desired profile to the plaster of Paris blocks. The profile used in the experiments was that given by the graph of a parabola, $y = x^2$, with 1 cm. as the unit. To keep the wax from adhering, the mold was covered with a sheet of waxed paper, and the hot wax poured on the paper.

When the block of wax was cut at the line a-b (Fig. 2), the two parts could be fitted together as is shown in Figure 3.

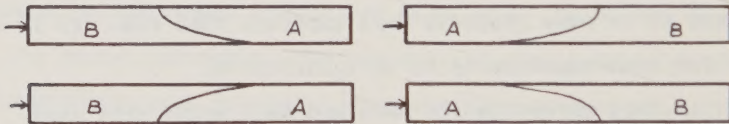


Fig. 3.--Manner of fitting blocks together

In this way the desired major or primary fracture was made. The fit obtained was close, but not tight; a little sand from the over-burden could sift down between the blocks. There were several possibilities for the material beneath the blocks. It could be of contracting rubber, or of yielding sand, or of some unyielding material over which the blocks could slide more or less freely. As it did not appear that significant differences would be found by systematically trying each of the three, a substratum of sand, which is, in a way, intermediate between the other two, was usu-

ally selected. This layer was made about two inches thick. In a few of the experiments, however, the blocks rested directly on the wooden bottom of the pressure box. The over-burden consisted of about two inches of sand just above the wax, next, about four inches of lead shot, and, on top of all, about four inches of iron bars.

Either block A or block B was used as the active block (Fig. 3), and the initial fracture was placed either convex up or concave up. Thus there were four series of experiments. The rate of deformation varied from about one inch of thrust per minute to about six inches per minute. In order to observe the progress of the deformation, the thrusting was stopped several times during an experiment and the over-burden removed. At the end of an experiment, structures which had developed were traced through the blocks by sectioning them.

The structures produced in the experiments using hard wax showed notable uniformity in certain major features. Other generalizations were readily drawn from the experiments in which soft wax was used. In some ways the hard and soft wax behaved similarly. Since the individual experiments included in a series showed considerable uniformity of behavior, it was thought better to give the results in summary form, rather than to describe each experiment individually. The results for each series are summarized in the paragraphs below accompanied by sketches from the experiments.

RESULTS OF EXPERIMENTS WITH HARD WAX

1. Importance of fracturing.

The structures produced in a block of hard wax by the thrusting showed a preponderance of fracturing over folding or mashing.

2. Position of fractures.

a. In the case in which the fault surface is concave up, as in Figure 4, fractures are developed in the lower part of the

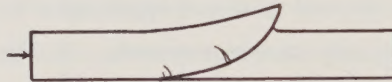


Fig. 4.--Fault surface concave up

active block. As the active block moves to the right, it rises on the passive block. Due to the weight of the over-burden, the active block is deformed so that it fits the upper surface of the lower block. Since the forward or right-hand part of the fault surface is more sharply curved than the left-hand part, the active block must bend more and more as it moves. This bending causes tension in the lower part of the active block, which in turn produces the cracks observed in this part of the block.

b. In the case in which the fault surface is convex up, as in Figure 5, fractures are developed in the upper part of the active block. The active block, in moving to the right, rises on the passive block and a gap tends to develop below its right-hand end, as is shown diagrammatically in Figure 6. The weight of the



Fig. 5.--Fault surface convex up

over-burden forces this end down, however, and a tension crack is developed in the upper part of the block where the tension caused by bending is greatest.



Fig. 6.--Gap under active block

3. Shape of fractures.

The tension cracks, formed in the above manner, begin as short fractures which are nearly straight and slightly inclined to the left away from the normal to the surface of the block as is shown in Figure 7. They develop further into curved or L-shaped



Fig. 7.--Examples of tension cracks formed in overthrusts

cracks invariably pointing backward toward the source of pressure. This peculiar but characteristic feature is probably caused by diagonal tension, to be described later in the section on "Effect of Bending in an Elastic Block."

4. Effect of stratification.

When the blocks are made of several layers (usually about six in these experiments), L-shaped fractures are much more common than curved fractures (Fig. 8). This is because the

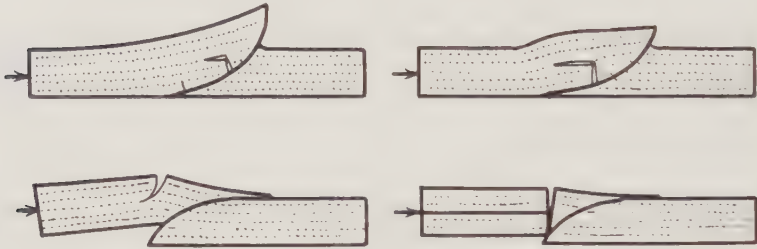


Fig. 8.--Structures produced in stratified blocks

bedding planes are important planes of weakness. When a block is bent by the thrusting, in the manner described above, some shearing occurs. Fractures which start across the bedding turn abruptly so as to continue along planes of weakness and shear between beds.

5. Underthrust vs. overthrust.

Though the active block is usually a little more fractured than the passive block, it seems that underthrusting is not essentially different in its results from overthrusting. The underthrust blocks are bent in the same way as the corresponding overthrust blocks, and similar fractures develop in the two cases. Thus Figures 9 and 10 correspond to Figures 5 and 4.



Fig. 9.--Underthrust with fault surface concave up



Fig. 10.--Underthrust with fault surface convex up

6. Effect of substratum.

In the experiments in which a substratum of sand was used, the toe of the under block was often caught by the sand and bent downward into it. This occurred in the cases shown in Figures 9 and 10.

7. Effect on angles.

Sharp angles tend to be rounded off. This is a minor effect due to abrasion; it occurs where two parts rub.

RESULTS OF EXPERIMENTS WITH SOFT WAX

1. Effect of material.

The use of soft wax resulted in a greater prominence of flow than when hard wax was used. Shear wedges were distinct, whereas they were not observed in the blocks of hard wax. With soft wax, zones of deformation were larger and more regular in outline, and fracturing was less simple or clear cut.

2. Effect of curvature of fault surface.

a. When the profile was concave up (Fig. 11), the upper



Fig. 11.--Fault surface concave up

block was bent, and the dip of its forward end steepened by the process outlined in paragraph 2a of the preceding section. There was not much overthrusting within the block, but wedges of shear were formed and horsts and grabens produced.

b. When the profile was convex up (Fig. 12), the most



Fig. 12.--Fault surface convex up

notable result was the production of small overthrust (and some-

times underthrust) slices at the forward end of the active block. This fracturing was due to the resistance and weight of the overburden acting on the thin front end of the active block as it rose and moved forward.

3. Nature and development of individual fractures.

The first noticeable effect was the appearance of wedges of shear, marked by horsts above and below (Fig. 13a). Next, the

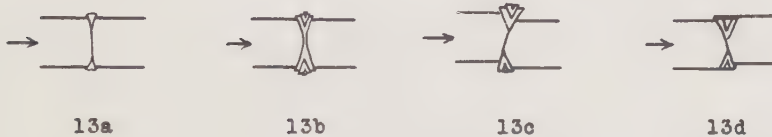


Fig. 13.--Development of fractures in soft wax

zone of deformation widened through the action of flowage and the formation of small inclined fractures outside the earlier ones (Fig. 13b). Finally a major fracture developed along the sides of two oppositely directed wedges of shear. As the diagram shows, the result might be either an overthrust (Fig. 13c) or an underthrust (Fig. 13d), depending on which sides of the zones of shear the fracture followed.

4. Order of appearance of fractures.

The first fractures appeared near the forward end of the active block; later fractures in the block formed farther back from this end. Movement often took place in an earlier fracture zone after a later fracture had appeared.

5. Influence of substratum.

With a yielding substratum of sand, the nose of the lower block was often caught and bent down. This effect was the same as that obtained with hard wax.

6. Overthrust vs. underthrust.

No significant difference was observed in the results obtained when pressure was applied to the foot-wall block (underthrust) instead of to the hanging-wall block (overthrust). Observation of the minor structures showed that an overthrust can pass into an underthrust along the strike of a zone of deformation. Figure 14 is a diagram of this phenomenon. As will be seen, the

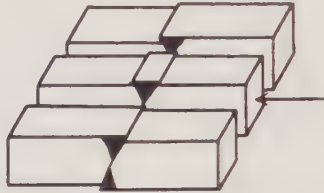


Fig. 14.--Diagram showing an underthrust passing into an overthrust along the strike of a zone of deformation.

zone of deformation (shaded) is a double wedge. On the near side of the block the fracture has run along those sides of the wedges which dip to the left, and the result is an underthrust. On the far side, the fracture dips to the right and the result is an overthrust.

RESULTS OF EXPERIMENTS WITH ALTERNATE
LAYERS OF HARD AND SOFT WAX

1. There was more shear between layers than was found in the preceding experiments with homogeneous material.

2. There was greater complexity in the shattering of the brittle layers, and less extensive development of wedges of shear in the soft layers, than occurred before. This was evidently due to the thinness of the layers.

3. The fracture system in any one layer often showed no relation to that in the other layers.

4. In general, the rigid layers behaved as in the experiments with hard wax only, and the weak layers as in those with soft wax. However, the notable independence of behavior of individual layers gave greater complexity to the entire structure.

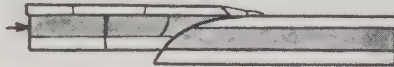


Fig. 15.--Fault surface convex up.
Soft wax shaded, hard wax white.



Fig. 16.--Fault surface concave up.
Soft wax shaded, hard wax white.

MOTION OF A RIGID BLOCK ON A PLANE SURFACE

In the mathematical approach to the mechanism of thrust faulting, a number of rather simple problems are here considered. By the solutions of these problems certain principles are made manifest which are important in the more complex problems of thrust faulting. The first problem considered is the influence of friction on the motion of a rigid, wedge-shaped block which is pushed up an inclined plane by a horizontal force. Though this is a very simple case, it is instructive to the student of faulting. The general principle involved was worked out long ago,¹ but the graphical presentation which follows should be more understandable and convincing to many geologists than the analytical method of earlier presentations.

The wedge-shaped block is shown in section in Figure 17.

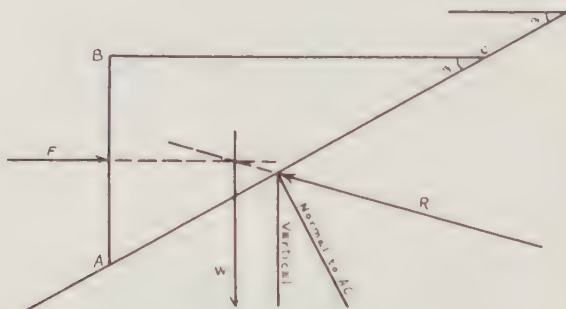


Fig. 17.--Profile of a block on an inclined plane showing the forces acting on the block.

¹Rev. O. Fisher, "On Faulting, Jointing, and Cleavage," Geological Magazine, June, 1884, p. 266.

For simplicity, the thickness of the block (dimension perpendicular to this section) is taken as unity. The upper surface, BC, is horizontal, and the left-hand surface, AB, is vertical. The plane on which the block rests has a dip denoted by α . In order to find the lower limit of the force necessary to push the block up the plane, assume that the block is on the point of moving. Any smaller push will not move the block, whereas any greater push will move it. The block is acted on by three forces. First, a horizontal force, F , acts on side AB. For simplicity, the point of application of F is taken midway between A and B. Second, is the weight of the block, W , acting vertically at its center of gravity. Third, the frictional resistance operates along its lower surface AC. When the block is on the point of moving, or is moving, the frictional resistance acts at an inclination to the normal which is equal to the angle of friction, ϵ . The resistance may be reduced to an equivalent single force, R , acting at the angle of friction which is an angle whose tangent is equal to the coefficient of friction, μ ; i.e., $\tan \epsilon = \mu$.

If the block is on the point of moving, it is in equilibrium under the action of these three forces, F , W , and R . According to elementary principles of mechanics, if a body is in equilibrium under the action of three forces, (1) the lines of the forces meet in a point, and (2) they form a closed triangle of forces. The force W is completely determined. Its line of action is vertical; its magnitude is the weight of the block; its point of application is the center of gravity of the body. The force F is partly determined to begin with. Its line of action is horizontal; its point of application is the mid-point of AB; its magnitude remains to be found. Only the inclination of the resistance, R , is known. This inclination from the normal to AC

is equal to the angle of friction.¹ The point of application of R is determined by the fact that the lines of action of the three forces meet in a point. Hence its line of action must pass through the known intersection of the lines of F and W (Fig. 17). The magnitudes of both F and R are determined by the fact that the three forces, F , W , and R , form a closed triangle of forces. In this triangle all the angles are known, since we know the inclinations of the lines of action of the forces and the magnitude of one side, W . Then, by elementary graphical principles, the magnitudes of the other two sides can be found. Thus the intersection of the lines of F and R gives their magnitudes.

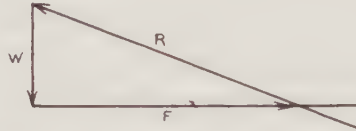


Fig. 18.--Triangle of forces showing intersection of F and R .

Figure 19 shows graphically the forces on a block where α , the dip of the fault plane, is 45° , and μ , the coefficient of friction, has the values indicated. In the case where $\mu = 1.0$, $\tan \epsilon = 1.0$, and $\epsilon = 45^\circ$. The angle which R makes with W is $\epsilon + \alpha$; in this case $45^\circ + 45^\circ = 90^\circ$. Hence R is perpendicular to W . But F is also perpendicular to W . Therefore F is parallel to R , and F and R can not intersect. This shows that under these conditions F can never be made large enough to move the block. It is evident that this will be true for any block when F is horizontal, if μ becomes large enough. This limiting condition is

¹Since R makes an angle ϵ with the normal to AC (Fig. 17), and AC makes an angle α with the horizontal BC , R makes an angle $\alpha + \epsilon$ with the vertical line of action of W , and an angle $90^\circ - (\alpha + \epsilon)$ with the horizontal line of action of F .

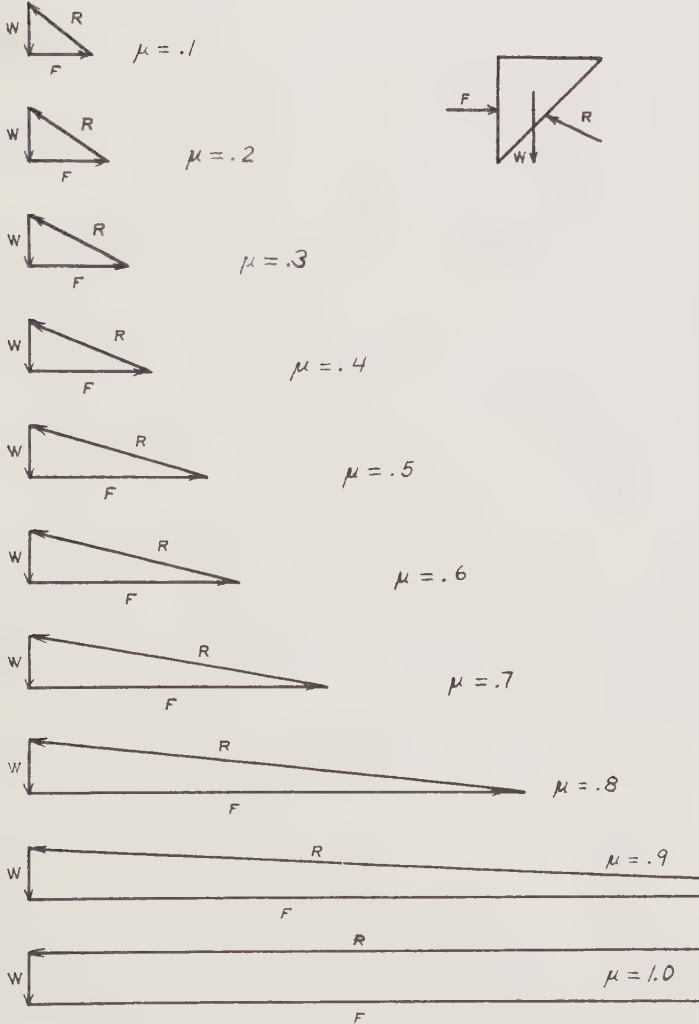


Fig. 19.--Triangles of forces on a block for which $\alpha = 45^\circ$ and μ has the values indicated.

reached when $\epsilon + \alpha = 90^\circ$, for then R is parallel to F , and, as explained above, the block cannot be moved. Figure 20 shows the values of μ and ϵ with the corresponding values of α at which this limiting condition is reached. For example, if μ , the coefficient of friction in a given case, is 0.6, then horizontal pressure cannot push a block up an incline with a dip, α , of 60° . In this case ϵ , the angle of friction corresponding to μ , is 30° .

The relative forces necessary to move blocks of equal mass, but with different values of α , are shown in Figure 21. The left-hand column shows the blocks in section, each section having the same area, while the right-hand columns show the corresponding triangles of force, one column for $\mu = 0.6$, the other for $\mu = 0.7$. The relative forces required to move such blocks are shown in Figure 22. Here four series of cases are illustrated for four different values of μ . It will be seen that for values of α below about 35° , a moderate change in μ does not necessitate a large change in the force, but for higher values of α , the contrary is true. Although these figures are for blocks of triangular section with the angle at one corner equal to the dip, α , of the plane (Fig. 17), the principles developed here apply to blocks of other shapes which could slide on the plane. Cylinders or other odd shapes which would roll or topple must, of course, be excluded.

Although it is true that for a given value of μ a smaller force is needed to move blocks with a low value of α than those with a high value, yet this smaller force must be applied to a smaller area, and the pressure may be greater. The relative sizes of the areas on which the forces are applied are shown in the left-hand columns of Figure 21. If these forces are each applied as a uniform pressure on the left-hand surfaces of the blocks, the

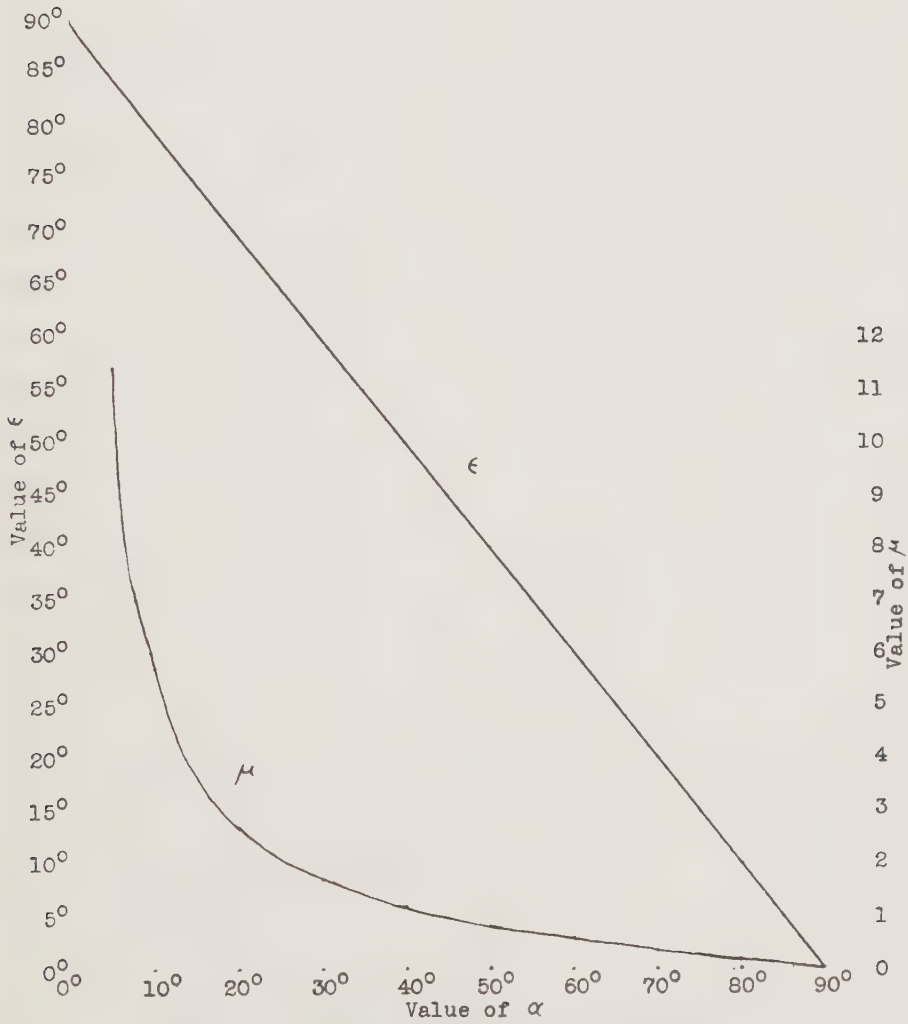


Fig. 20.--Values of μ and ϵ which, for a given value of α , make motion of the block impossible.

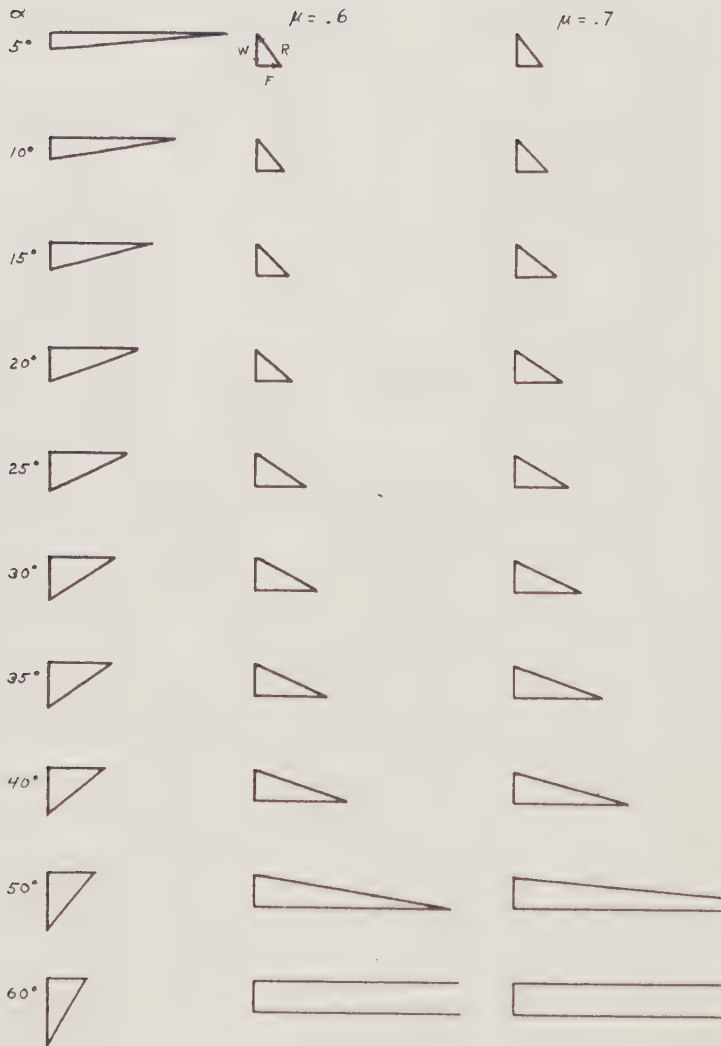


Fig. 21.--Relative forces required to push unit blocks up planes of various inclinations.

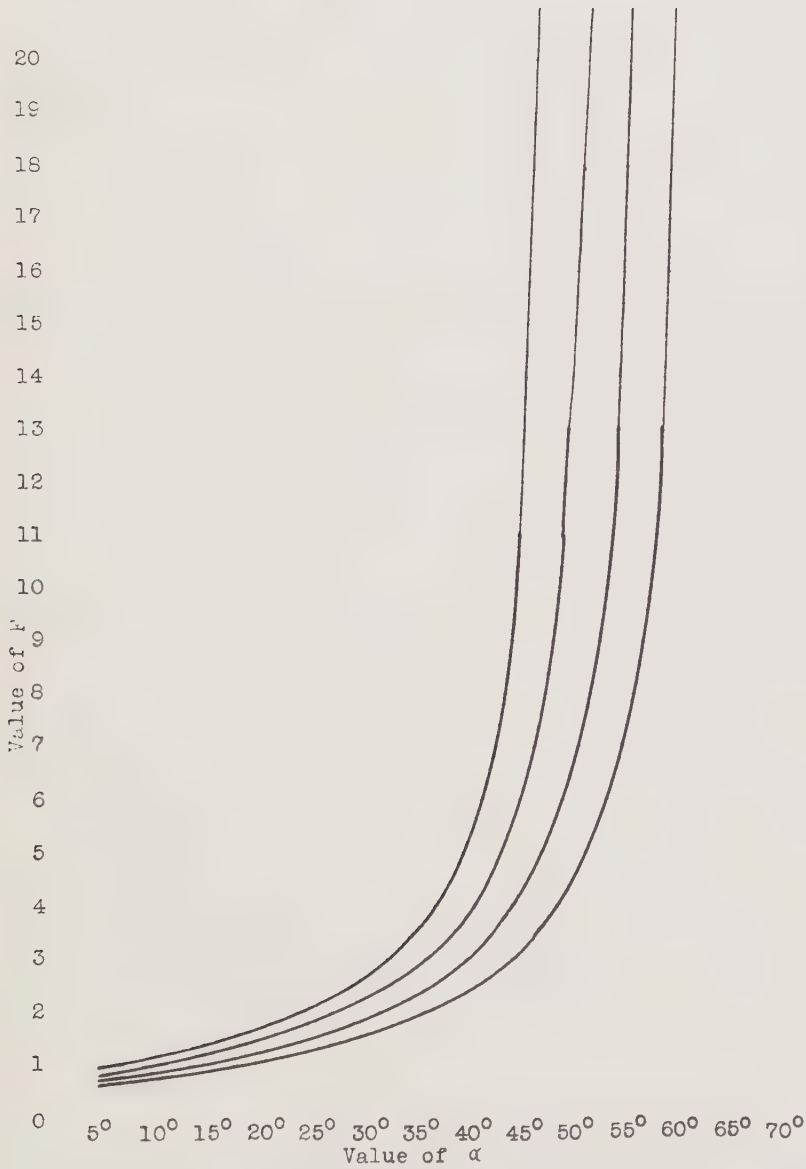


Fig. 22.--Relative values of force required to push a unit block up planes of various inclinations.

relative values of these pressures are obtained by dividing each force by the corresponding area. These pressures are shown in Figure 23. It will be seen that there is not a great difference in the pressures required to move blocks up inclines of from 5° to 35° , but that much higher pressures are required in the cases of steep or very gentle inclines.

The relative amount of work involved in moving blocks a unit distance up inclines with various values of α is shown in Figure 24. The distance in each case is measured along the incline; it is the slip, rather than the throw or heave. Figure 25 shows the relative amount of work done against friction alone. Figure 26 indicates what percentage of the total work is represented by the work done against friction. This percentage is low for angles of 30° to 35° , and it increases for lower and higher angles of dip. In an actual thrust fault the formation of gouge, slickensides, drag folds, etc. along or near the fault plane are no doubt related to the work done against friction, but it is, of course, impossible to express the relations quantitatively.

It may be concluded from this analysis that, although horizontal forces can move fault blocks up moderate inclines with relative ease, motion on steeper inclines quickly becomes more difficult and finally impossible. This does not mean, however, that high-angle thrusts are impossible. When such are found, non-rigid behavior of the fault block, or operation of the force at some angle other than the horizontal should be suspected.

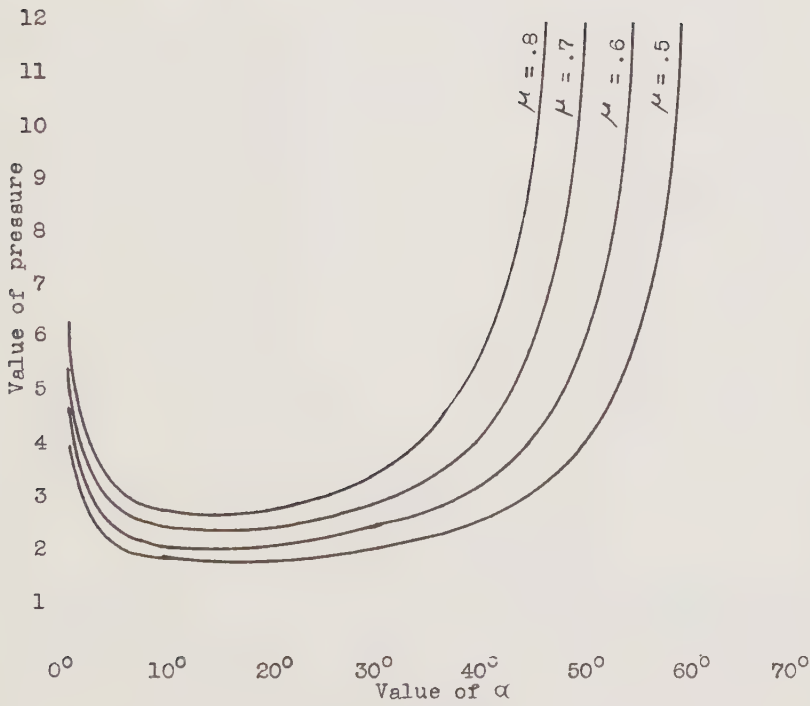


Fig. 23.--Relative pressures required to move unit blocks up planes of various inclinations.

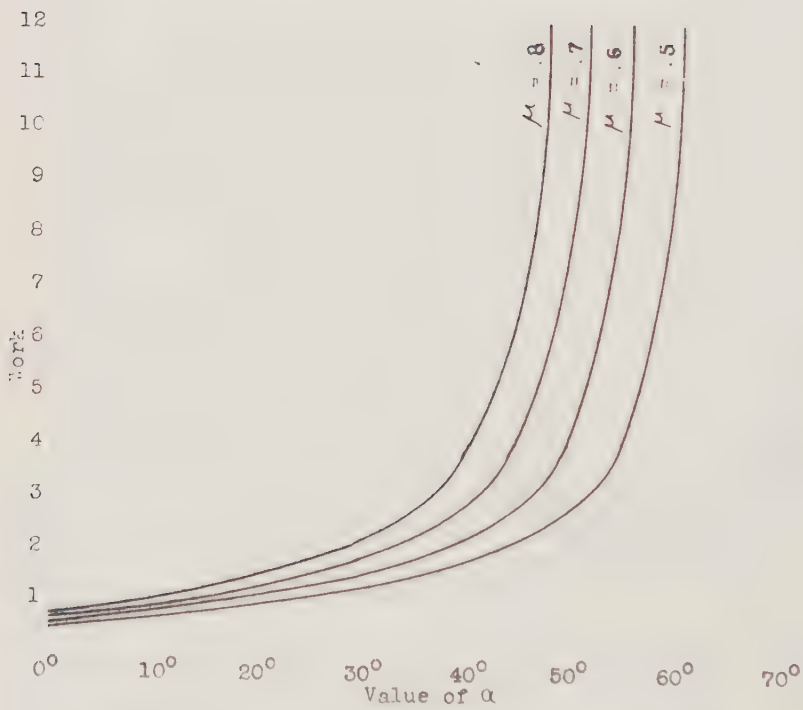


Fig. 24.--Relative amounts of total work done in pushing a block up planes of various inclinations.

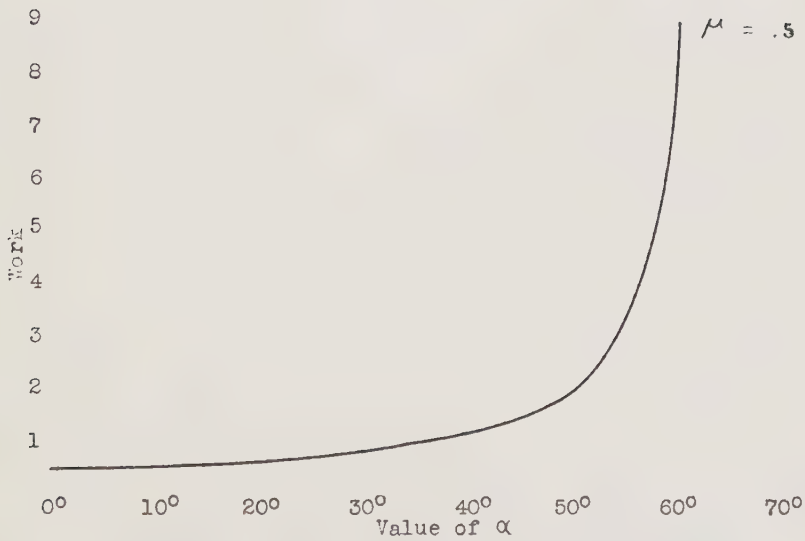


Fig. 25.--Relative amounts of work done against friction in pushing a block up planes of various inclinations.

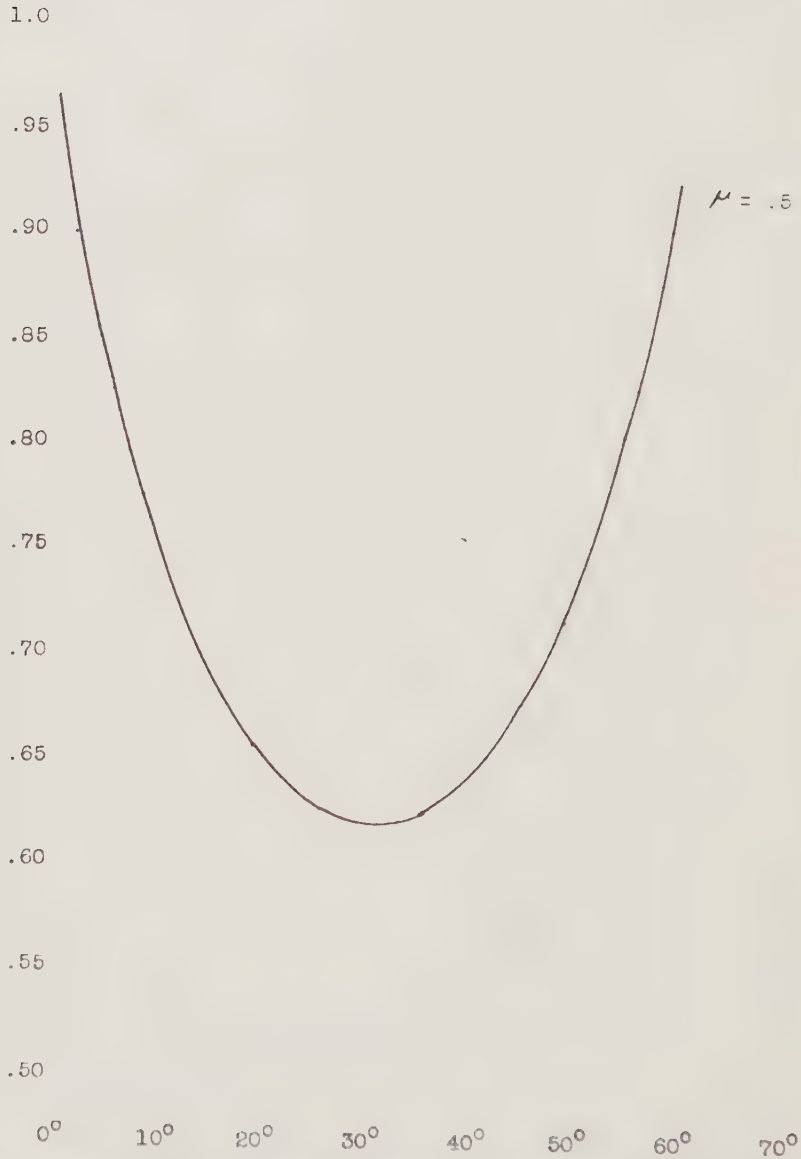


Fig. 26.--Ratio of work done against friction to total work done in pushing a block up planes of various inclinations.

MOTION OF A RIGID BLOCK ON A CURVED SURFACE

In the preceding section of this paper the block was considered to have moved on a plane surface. In this section we will consider the force required to move a block on a surface having a curved profile. For convenience we will imagine that the block is made up of a number of thin vertical slices, and will further assume that there is no friction between adjoining slices.

With the aid of these assumptions, the bending forces which would otherwise be involved may be disregarded. As the block moves, the slices adjust themselves to the curved underlying surface by relative motion among themselves. Necessarily the result obtained by this method is only an approximation; it most closely approaches the real result where the movement of the block is very small, for then the bending of the block is also very small. The method adopted is to find an expression for the force necessary to move any one slice by itself. The force required to move the whole block is the sum of the forces required to move the individual slices. The sum of the forces required to move an infinite number of infinitely thin slices is found by methods of the integral calculus.

The block is shown in profile in Figure 27. The surface on which the block moves has a profile represented by the curve $y = f(x)$. The profile of the top of the block is assumed to be straight initially, parallel to the x -axis, and at a distance b from this axis. The dip of the surface on which the shaded slice rests is represented by α , which is also the angle between the

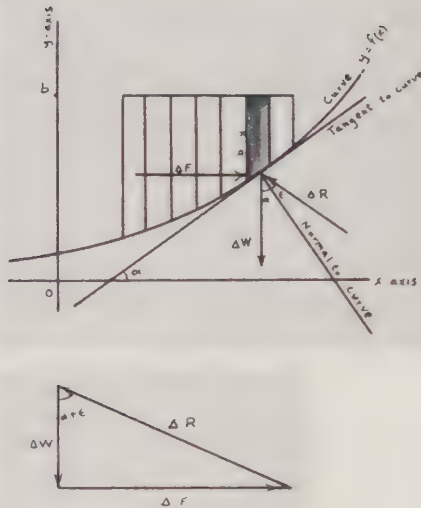


Fig. 27.--Diagram showing the profile of a block on a curved surface. The block is divided into slices of thickness Δx , and the forces acting on a single (shaded) slice are represented both in the profile and in the triangle of forces below.

x-axis and the tangent to the curve, $y = f(x)$, at the part of the curve below the shaded slice. ΔF is the lower limit of the horizontal force needed to move the slice; ΔW is the weight of the slice; and ΔR is the frictional resistance of the underlying surface. The distance from the x-axis to the base of the slice is y . Hence the height of the slice above the curve is $b - y$. The thickness of the slice is Δx , and the third dimension is taken as unity. The density is denoted by δ . ϵ is the angle of friction, and μ is the coefficient of friction for the surface ($\tan \epsilon = \mu$). It is assumed that the material of the slices is rigid. The situation for a single slice is thus like that already considered

for a block pushed up an inclined plane.

As Figure 27 shows, ΔR makes an angle of $\alpha + \epsilon$ with ΔW . From the triangle of forces,

$$\Delta F = \Delta W \tan(\alpha + \epsilon).$$

Substituting for ΔW its equal, $\delta(b - y)\Delta x$,

$$\Delta F = \delta(b - y) \tan(\alpha + \epsilon) \Delta x.$$

Expanding the expression $\tan(\alpha + \epsilon)$,

$$\Delta F = \delta(b - y) \frac{\tan \alpha + \tan \epsilon}{1 - \tan \alpha \tan \epsilon} \Delta x.$$

By the differential calculus, $\tan \alpha = \frac{dy}{dx}$, and hence

$$\Delta F = \delta(b - y) \frac{\frac{dy}{dx} + \mu}{1 - \mu \frac{dy}{dx}} \Delta x.$$

Taking the sum of the F for all the slices by the integral calculus,

$$F = \int_{x_1}^{x_2} \delta(b - y) \frac{\frac{dy}{dx} + \mu}{1 - \mu \frac{dy}{dx}} dx.$$

This expression may be roughly translated to mean: The force required to move that portion of the block between the lines $x = x_1$, and $x = x_2$,¹ is the sum of the forces required to move an infinite number of infinitely thin slices of density δ , and of length $(b - y)$, with the slope of the base $\frac{dy}{dx}$, and the coefficient of friction at the base μ . With the help of this formula, the

¹In the cases given below, these lines are $x = 1$ and $x = 4$.

force required to move a block on a surface which is convex up can be compared with the force required to move a block of equal mass on a surface which is concave up, or which is a plane.

Case I. Force required to push a block up a surface which is convex up.

The block (shaded) and surface are shown in profile in Figure 28. In this and the following parts of this section of

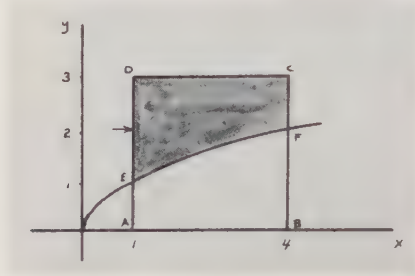


Fig. 28.--Profile of block on a convex surface

the paper, the density of the block and coefficient of friction are taken as unity. The mass of any block is proportional to the area of its profile. If the areas of the profiles of the blocks are equal, then their masses are equal. The profile of the underlying surface is the curve $x = y^2$. Hence $dx = 2ydy$. In this and the following cases the length of the block is taken to be three. The height of the upper surface of this block above the x -axis is three. The area, A_1 , of the profile of the block is equal to the area of the rectangle ABCD minus the area under the curve ABFE.

$$A_1 = 9 - \int_1^4 y dx = 9 - \int_1^2 2y^2 dy = 9 - \left[\frac{2y^3}{3} \right]_1^2 = 9 - \left(\frac{16}{3} - \frac{2}{3} \right) = \frac{13}{3}.$$

Height of each slice of the block above the curve $= b - y = 3 - y$.

Slope of the profile of the bottom of the block $= \frac{dy}{dx} = \frac{1}{2y}$. The

dip of the underlying surface is 27° at E and 14° at F.

$$\begin{aligned}
 F_1 &= \int_1^4 (3 - y) \frac{\frac{1}{2y} + 1}{1 - 2y} dx = 3 \int_1^2 \frac{2y + 1}{2y - 1} 2y dy - \int_1^2 y \frac{2y + 1}{2y - 1} 2y dy \\
 &= 3 \int_1^2 \frac{4y^2 + 2y}{2y - 1} dy - \int_1^2 \frac{4y^3 + 2y^2}{2y - 1} dy \\
 &= 3 \int_1^2 2y + 2 + \frac{2}{2y - 1} dy - \int_1^2 2y^2 + 2y + 1 + \frac{1}{2y - 1} dy \\
 &= 3 \left[y^2 + 2y + \log(2y - 1) \right]_1^2 - \left[\frac{2y^3}{3} + y^2 + y + \frac{1}{2} \log(2y - 1) \right]_1^2 \\
 &= 3 \left[4 + 4 + \log 3 - 1 - 2 - \log 1 \right] \\
 &\quad - \left[\frac{16}{3} + 4 + 2 + \frac{1}{2} \log 3 - \frac{2}{3} - 1 - 1 - \frac{1}{2} \log 1 \right] \\
 &= 3 [6.10] - [9.22] = 9.08.^1
 \end{aligned}$$

Case II. Force required to push a block up a plane surface.

The block is shown in profile in Figure 29. As in Case I, the length of the block is three, and the difference between the

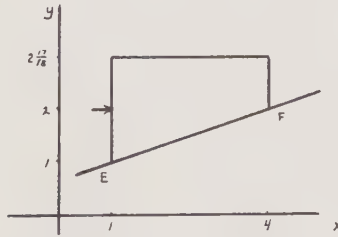


Fig. 29.--Profile of block on a plane surface

height of E and of F above the x-axis is one. The mass is equal to that of the first block, for dimensions are chosen which make

¹The forces in these cases are relative. The unit of force would depend on the units of length and of weight.

the areas of the profiles equal. The dip of the underlying surface is $18^{\circ}26'$.

$$A_2 = 3 \cdot \frac{1}{2} \left(\frac{17}{18} + \frac{17}{18} \right) = \frac{3 \cdot (35 + 17)}{2 \cdot 18} = \frac{3 \cdot 52}{2 \cdot 18} = \frac{13}{3}.$$

$$F_2 = W \tan(\alpha + \epsilon) = W \frac{\tan \alpha + \tan \epsilon}{1 - \tan \alpha \tan \epsilon} = \frac{13 \cdot \frac{1}{3} + 1}{1 - \frac{1}{3}} = \frac{13}{3} \cdot 2 = \frac{26}{3} = 8.67.$$

Case III. Force required to push a block up a surface which is concave up.

The block (shaded) and surface are shown in profile in Figure 30. As in the two preceding cases, the length of the block

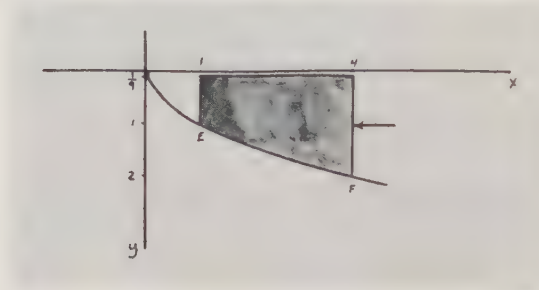


Fig. 30.--Profile of block on a concave surface

is taken to be three, and the difference in the elevation of E and F with respect to the x-axis is one. The mass is made equal to that of the other two blocks. The profile of the surface, $x = y^2$, in this case is exactly that of the first example turned upside down. In other words the segment EF in Figure 28 will, if turned upside down, coincide with segment EF in Figure 30. It will be noted in Figure 30 that the positive direction of the y-axis is down instead of up. The height of each slice of the block is $y - \frac{1}{9}$.

$$A_3 = \int_1^4 y dx - 3 \cdot \frac{1}{9} = \int_1^2 2y^2 dy - \frac{1}{3} = \left[\frac{2y^3}{3} \right]_1^2 - \frac{1}{3} = \frac{2}{3}(8 - 1) - \frac{1}{3} = \frac{13}{3}.$$

The dip of the underlying surface varies from 14 at F to 27 at E.

$$\begin{aligned} F_3 &= \int_1^4 \left(y - \frac{1}{9} \right) \frac{\frac{1}{2y} + 1}{1 - \frac{1}{2y}} dx = \int_1^2 \left(y - \frac{1}{9} \right) \frac{2y + 1}{2y - 1} 2y dy \\ &= \int_1^2 \frac{4y^3 + 2y^2}{2y - 1} dy - \frac{1}{9} \int_1^2 \frac{4y^2 + 2y}{2y - 1} dy \\ &= \int_1^2 \left(2y^2 + 2y + 1 + \frac{1}{2y - 1} \right) dy - \frac{1}{9} \int_1^2 \left(2y + 2 + \frac{2}{2y - 1} \right) dy \\ &= \left[\frac{2}{3}y^3 + y^2 + y + \frac{1}{2} \log(2y - 1) \right]_1^2 - \frac{1}{9} \left[y^2 + 2y + \log(2y - 1) \right]_1^2 \\ &= \left[\frac{16}{3} + 4 + 2 + .55 - \frac{2}{3} - 1 - 1 \right] - \frac{1}{9} [4 + 4 + 1.1 - 1 - 2] \\ &= 9.22 - \frac{1}{9} [6.10] = 8.54. \end{aligned}$$

Conclusion:

Force required when bottom is convex up: $F_1 = 9.08$.

Force required when bottom is a plane: $F_2 = 8.67$.

Force required when bottom is concave up: $F_3 = 8.54$.

These results indicate that for blocks of equal mass, those whose bottom is concave up are more easily moved than comparable blocks with bottoms convex up. This result is readily seen intuitively; for in the former case the longer and heavier slices move on a gentle incline, while in the latter case the longer slices move on a steeper incline.

ELASTIC MOTION OF A FLAT PLATE

In this section of the paper the motion of an elastic, rather than a rigid, block is considered. For simplicity, a rectangular plate lying on a horizontal plane is selected (see Fig. 31). To make the picture a bit more like that of a thrust fault, it is assumed that the block originally lay unstrained with its ends at A' and B . While end B is held fixed, the other end is pushed to A by pressure from the left. Now the condition of the block is similar to that of a strained fault block. Next, end A is held fixed while the compressed block is released at B . The resilient motion of the block is similar to that of a fault block during faulting.

It is assumed that the plate expands or contracts uniformly and as a whole. From symmetry the center of mass is midway between the ends of the block. The plate, if not strained, would extend from A to C ($A'B = AC$), with its center of mass at M_C . M_B and M_D are the positions of the center of mass when the free end is at B and D respectively. The motion of the center of mass of the released plate is found in terms of its distance, s , from M_C .

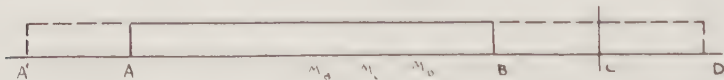


Fig. 31.--Profile of elastic plate

s is taken to be positive when to the right of M_C , and negative when to the left. The motion of the free end of the block is

double the motion of the center of mass, and the distance, s , of the free end from point C is double the distance, s , of the center of mass from M_C . The motion of the center of mass is calculated for two cases, (1) with no friction between the plate and the plane surface on which it rests, and (2) with friction between the plate and the underlying surface.

Case I. With no friction between plate and surface.

F = force acting on the center of mass.

s = displacement of the center of mass from M_C .

$v = \frac{ds}{dt}$ = velocity of the center of mass.

$a = \frac{d^2s}{dt^2}$ = acceleration of the center of mass.

m = mass of the plate.

t = time, measured from the instant the block is released.

b = maximum displacement of the center of mass.

k^2 = constant of proportionality for Hooke's law.

From Hooke's law, the restoring force is proportional to the strain, $F = -k^2s$. The minus sign is used because the force is positive in direction when the displacement is negative. By definition,

$$F = ma = m \frac{d^2s}{dt^2}.$$

Hence,

$$m \frac{d^2s}{dt^2} = -k^2s.$$

Since $v = \frac{ds}{dt}$,

$$\frac{d^2s}{dt^2} = \frac{dv}{dt} = \frac{dv}{ds} \frac{ds}{dt} = v \frac{dv}{ds}.$$

Substituting $v \frac{dv}{ds}$ for $\frac{d^2s}{dt^2}$,

$$mv \frac{dv}{ds} = -k^2 s.$$

Separating the variables and integrating:

$$\frac{1}{2}mv^2 = -\frac{1}{2}k^2 s^2 + C_1.$$

To evaluate the constant of integration: when the plate is released, $v = 0$ and $s = -b$. Hence,

$$0 = -\frac{1}{2}k^2 b^2 + C_1.$$

Substituting $\frac{1}{2}k^2 b^2$ for C_1 ,

$$(1) \quad v = \pm \sqrt{-\frac{k^2 s^2}{m} + \frac{k^2 b^2}{m}}.$$

Substituting $\frac{ds}{dt}$ for v ,

$$\frac{ds}{dt} = \sqrt{-\frac{k^2 s^2}{m} + \frac{k^2 b^2}{m}}; \quad \frac{ds}{\frac{k}{m} b^2 - s^2} = dt.$$

$$\arcsin \frac{s}{b} = \frac{kt}{m} + C_2$$

$$s = b \sin \frac{kt}{m} + C_2.$$

To evaluate C_2 , when $t = 0$, $s = -b$. Hence, $b \sin C_2 = -b$,
 $\sin C_2 = -1$,

$$C_2 = \frac{3\pi}{2} \text{ or } C_2 = -\frac{\pi}{2}.$$

Substituting $-\frac{\pi}{2}$ for C_2 ,

$$(2) \quad s = b \sin\left(\frac{kt}{m} - \frac{\pi}{2}\right).$$

Since the distance of the free end of the block from point C is $2s$, or double the distance of the center of mass from M_C , equation 2 shows, from the periodicity of the sine function, that the free end of the plate oscillates in simple harmonic motion from B, where $CB = -2b$, to D, where $CD = 2b$, and back. The period is equal to $\frac{2\pi}{k}\sqrt{m}$. Equation 1 shows that the velocity of the end of the plate is a maximum, $\frac{2kb}{\sqrt{m}}$, when $s = 0$, i.e., when the end of the plate is at C, and the velocity is zero when $s = \pm b$, i.e., when the end of the plate is at B or D.

Case II. With friction between plate and surface.

It is assumed that the coefficient of friction, μ , is independent of the velocity. The motion caused by the restoring force, $F = -k^2s$, is opposed by the force of friction, $F_f = m$. The method used is similar to that in Case I.

$$m \frac{d^2s}{dt^2} = -k^2s - m.$$

This equation holds only for motion in the positive direction.

Substituting $v \frac{dv}{ds}$ for $\frac{d^2s}{dt^2}$,

$$mv \frac{dv}{ds} = -k^2s - m.$$

Integrating,

$$\frac{1}{2}v^2 = -\frac{k^2s^2}{2m} - s + C_1.$$

Since $v = 0$ when $s = -b$,

$$0 = -\frac{k^2b^2}{m} + 2b + 2C_1, \quad C_1 = \frac{k^2b^2}{2m} - b.$$

$$v = \sqrt{-\frac{k^2s^2}{m} - 2s + \frac{k^2b^2}{m} - 2b}.$$

The center of mass ceases to move to the right when it reaches the point $s = b - \frac{2\mu m}{k^2}$, for this value for s makes $v = 0$ in the equation above. The free end of the plate is, of course, at twice this distance from the point C. Motion to the left will now begin only if the restoring force exceeds that of the frictional resistance, i.e., if $k^2 s > \mu m$; otherwise the plate will be held fast by friction. If motion to the left does occur, the process will be similar to that described for motion to the right. The plate may thus come to rest when strained by either tension or compression. It is highly unlikely that it will stop when in the position of no strain. If the analogy with thrust faulting is at all close, it follows that faulting may result in tensile stresses in the fault block, and, in any case, the block will probably still be considerably strained when it comes to rest. An important factor in the actual process is that the coefficient of friction for starting is greater than that for the same surfaces in motion. It follows, therefore, that once a body starts to move, it will move much farther than it would if the coefficient of friction did not become less after starting. Furthermore it is improbable that fault blocks expand or contract uniformly and as a whole. Certain parts of the block might move while others did not, and, when the block came to rest, some parts might be under tensile stress, whereas other parts were under compression.

EFFECT OF BENDING IN AN ELASTIC BLOCK

Let us next consider the forces which are involved in the bending of a block. These forces are important in thrust faulting, for, as a fault block moves, it adjusts itself to the adjacent block so that no large hollows form along the fault plane. Such adjustment is necessary unless the profile of the fault plane is either a straight line or an arc of a circle, ideal cases which could not occur in nature. Any part of the overlying block which is over an incipient hollow will bend down. The situation is comparable to the case of a beam supported at its ends while its middle portion is bent downward. An analysis of the behavior of such a beam gives an approximation to the behavior of a fault block. The reader is referred to books on the strength of materials for an extensive treatment of the subject; only a brief summary of the results pertinent to this study is given here.

The horizontal shearing unit-stress at any point in a bent beam is equal to the vertical shearing unit-stress. The value of this shear is given, in terms of the external forces and dimensions of the beam, by the equation

$$s_s = \frac{V}{It} \int_{y_0}^c y da.$$

In this equation, s_s is the horizontal shearing unit-stress in a cross section for which the vertical shear¹ is V , and at a point

¹The vertical shear for a section of a beam is the magnitude of the resultant of the forces that lie on one side of the section.

whose distance from the neutral axis is y_0 ; t is the thickness of the beam at the distance y_0 from the neutral axis; and I is the moment of inertia of the whole cross section of the beam about the neutral axis. The expression $\int_{y_0}^c y da$ is the first moment about the neutral axis of that part of the cross sectional area of the beam between the plane on which the horizontal shearing unit-stress, s_s , occurs and the outer face of the beam. The distribution of shear can be most readily visualized with the help of a graph. Figure 32 shows the variation of the horizontal

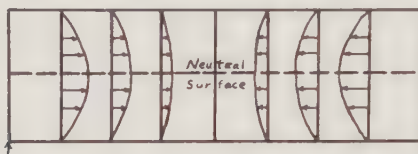


Fig. 32.--Horizontal shearing unit-stress in a simple beam

shearing unit-stress at seven sections of a simple beam subjected to a uniform load. The lengths of the arrows indicate the magnitude of the stresses. It will be seen that in any one cross section the unit-shear is greatest at the intersection of the section with the neutral surface, and decreases to zero at the upper and lower surfaces of the beam. Also the unit-shear is zero in the section which passes through the midpoint of the beam, and increases in the sections toward the ends of the beam. The fibers¹ of the beam are affected by tension or compression. The unit-stress is given by the formula $s = M_T \frac{c}{I}$, where s is the tensile or compressive unit-stress on a fiber at the distance c

¹A fiber is an imaginary rod having a small or elementary cross-sectional area and extending the length of the beam parallel to the axis of the beam.

from the neutral axis, M_r is the resisting moment¹ (= bending moment), and I the moment of inertia. The magnitude of s thus depends on the two variables c and M_r . The relation of s to c in a vertical cross section AB of the beam is shown graphically in Figure 33, from which it is apparent that the intensity of stress

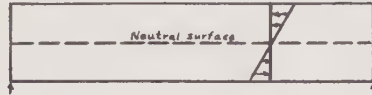


Fig. 33.--Normal unit-stress on a cross section (AB) of a beam.

(represented by the lengths of the arrows) varies directly as the distance from the neutral axis. The value of M_r for a uniformly loaded beam supported at the ends is represented in Figure 34 by

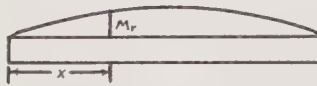


Fig. 34.--Graph of the resisting moment, M_r , in a uniformly loaded beam. The value of M_r is indicated by the height of the curve above the beam.

the height of the curve above the beam. Algebraically, $M_r = \frac{W}{2}x - \frac{wx^2}{2}$, where W = total weight, w = weight per unit length, and x = distance from the left-hand end of the beam.

At any point in a section of the beam there are thus, in general, both normal and shearing stresses. The normal unit-stress, s , on any vertical cross section (AB in Fig. 35) of the beam, as given by the formula above, is greatest at the outermost fiber. Since the shearing stress, s_s , at the outermost fiber

¹The resisting moment at a section of a beam is the algebraic sum of the moments of the stresses acting on the section about a line in the section.

is zero, the stress s is the maximum normal stress at this point of the beam. At a point, P in Figure 35, in the same section, but



Fig. 35.--Diagram of the forces at a point, P, in a vertical section, AB, of a simple beam. The force normal to the section is s ; the shearing force is s_s ; and the diagonal tension is s' .

nearer to the neutral axis, the normal stress on the vertical plane is given by the same equation, in which c is the distance from the neutral axis to the point P, but this normal unit-stress is not the maximum normal unit-stress at the point, since it is combined with shearing stresses on the vertical and horizontal planes. The resulting maximum normal unit-stress is $s' = \frac{1}{2}s + \frac{1}{2}\sqrt{s^2 + 4s_s^2}$, in which s , and hence s' , are tensile stresses at the point P. The plane, ab , on which the stress s' occurs is inclined to the cross section AB on which the stress s occurs. S' is frequently called the diagonal tension.

At any point¹ in a bent beam (or any other stressed body), a set of three mutually perpendicular planes can be found, on each of which the stress is purely normal. These planes are called the principal planes of stress for the point considered, and the corresponding stresses are called the principal stresses. To visualize this principle, imagine that each point in the body is a very small cube. It is possible to rotate any one of these

¹The statements in this paragraph are strictly true only when the size of the planes or cubes approaches zero.

cubes into a position such that only normal stresses act on its faces. In this position there are no shearing stresses on any of the faces. These particular normal stresses are the principal stresses at the point. For other positions of each tiny cube, both normal and shearing stresses will, in general, act on its faces. Also, the magnitude and direction of the principal stresses will vary from point to point in the body.

The manner in which the directions of the principal stresses vary in a simple beam is shown in Figure 36. The diagonal stresses vary in a simple beam is shown in Figure 36. The diagonal

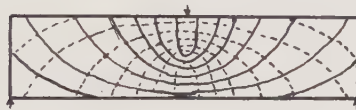


Fig. 36.--Directions of the principal stresses in a simple beam. The full lines give the directions of the tensile stresses and dotted lines the compressive stresses. The two sets are mutually perpendicular.

tension mentioned above is, of course, a principal stress, and the full lines give its direction. Cracks due to diagonal tension tend to parallel the dotted lines. The cracks obtained in the experiments described in the first part of this paper appear to have followed these lines of direction of principal stress. It seems reasonable that they are due to diagonal tension set up when the block is partially supported, as is shown in Figure 37.



Fig. 37.--Manner in which diagonal tension is produced

CONCLUSION

Both the experimental and mathematical results of this paper indicate that tension is important in thrust faulting. Though tension has been generally considered as acting mainly during a period of relaxation following active thrust faulting, it is evidently also important during the faulting. Tensional stresses may result from bending of the fault block or from the elongation of the block by elastic forces. Bending may prove to be important in many thrust faults; some of the results set forth in this paper may be useful in studying such faults in the field. Finally, the quantitative study of the forces involved in faulting is a part of the important problem of the energy changes involved in geological processes, a problem which will undoubtedly receive more and more attention as the study of geological processes becomes more exact.

